

Mathematical Basis of 1/f Fluctuations

Toshimitsu Musha

Brain Functions Lab., Inc.

4219-3, W301 TokyoTech YVP, Nagatsuta Midoriku, Yokohama, Japan 226-8510

E-mail: musha@bfl.co.jp

Classification: 1) Theory of 1/f fluctuations

Oral presentation

Our previous reports (Musha, and Tacano and Musha, Plenary Lectures in ICNF-2003, Prague; T.Musha and M.Tacano, Physca A 346 (2005) 339-346) discussed a theoretical background for generation of 1/f fluctuations based on Hamiltonian formulation of coupled harmonic oscillators. The result was in good agreement qualitatively as well as quantitatively with observed conductance fluctuations of semiconductors which had been prepared as carefully as possible. The important part of the discussion about the effect of nonlinear coupling of oscillators in the previous report was bridged by an intuitive speculation which is essentially identical to the idea of an equivalent circuit composition which was presented by Musha at the first conference of this series of ICNF in 1977(T.Musha, Proc. Symp. on 1/f Fluctuations, pp. 49-54, 1977, Tokyo). We now succeeded in replacing this intuitive speculation part by mathematical manipulations, and the whole story has been reformed in a consistent mathematical form. Equation of motion of nonlinearly coupled harmonic oscillators in equilibrium is solved based on the following Hamiltonian:

$$H = \hbar\omega a a^\dagger + \sum_j \hbar\omega_j b_j b_j^\dagger + \sum_j (k_j a b_j^\dagger + k_j^* a^\dagger b_j) + \sum_{jl} (k_j k_l a a b_j^\dagger b_l^\dagger + k_j^* k_l^* a^\dagger a^\dagger b_j b_l)$$

Where a and b 's are mode operators in which a refers to a target oscillator and b 's to other oscillators. All of these modes are supposed to have identical self-correlation functions. Without nonlinear term (the last term in H) each mode has a simple decay function $\exp(-\gamma t)$ where γ is given in terms of k_j^2 . The result is the same as what we have derived semi-objectively as well as semi-subjectively in our report in Physica A. Tacano examined velocity fluctuations of Brownian particles suspended in water without success in observing 1/f spectrum. Judging from observations that have been reported so far in various phenomena including electric conductivity of solids and liquids, phonon number fluctuations in a quartz single crystal, density fluctuations of water etc., we speculate that any physical entities described in terms of coupled modes or obeying Bose-Einstein statistics are subject to 1/f fluctuations while a collection of completely ideal-particle-like objects or those obeying Fermi-Dirac statistics are not.